

USN

--	--	--	--	--	--	--	--	--	--

15MATDIP31

Third Semester B.E. Degree Examination, July/August 2022

Additional Mathematics - I

Time: 3 hrs.

Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. Express $\frac{(2-3i)(2+i)^2}{1+i}$ in the form of $x + iy$. (06 Marks)
- b. If $x + \frac{1}{x} = 2 \cos \alpha$ then prove that $x^n + \frac{1}{x^n} = 2 \cos n\alpha$. (05 Marks)
- c. Find the cosine of the angle between the vectors $\vec{a} = 5\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} - 3\hat{j} + 6\hat{k}$. (05 Marks)

OR

- 2 a. Find the Fourth roots of $1 - i\sqrt{3}$ and represent them on an Argand plane. (06 Marks)
- b. Show that the vectors $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{b} = 2\hat{i} + \hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + 4\hat{j} - \hat{k}$ are co-planar. (05 Marks)
- c. Prove that $[\vec{a} + \vec{b}, \vec{b} + \vec{c}, \vec{c} + \vec{a}] = 2[\vec{a}, \vec{b}, \vec{c}]$. (05 Marks)

Module-2

- 3 a. Obtain the n^{th} derivative of $e^{ax} \cos(bx + c)$. (06 Marks)
- b. Show that the curves $r = a(1 + \cos\theta)$ and $r = a(1 - \cos\theta)$ are orthogonal. (05 Marks)
- c. If $u = x(1-y)$, $v = xy$ find the Jacobians $J = \frac{\partial(u,v)}{\partial(x,y)}$ and $J' = \frac{\partial(x,y)}{\partial(u,v)}$. (05 Marks)

OR

- 4 a. If $y = a \cos(\log x) + b \sin(\log x)$, prove that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$. (06 Marks)
- b. If $u = \sin^{-1}\left(\frac{x^3 - y^3}{x - y}\right)$, show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 2 \tan u$. (05 Marks)
- c. If $z = xy^2 + x^2y$, where $x = at^2$, $y = 2at$. Find $\frac{dz}{dt}$. (05 Marks)

Module-3

- 5 a. Evaluate $\int_0^\pi x \sin^6 x \, dx$. (06 Marks)
- b. Evaluate $\int_0^1 \int_0^1 \frac{dx dy}{\sqrt{(1-x^2)(1-y^2)}}$. (05 Marks)
- c. Evaluate $\int_0^1 \int_0^1 \int_0^1 (x+y+z) dx dy dz$. (05 Marks)

OR

- 6 a. Evaluate $\int_0^1 x^5 (1-x^2)^{\frac{5}{2}} x \, dx$. (06 Marks)
- b. Evaluate $\int_0^{2a} \int_0^{\frac{x^2}{4a}} xy \, dy \, dx$. (05 Marks)
- c. Evaluate $\int_0^1 \int_0^1 \int_0^y xyz \, dx \, dy \, dz$. (05 Marks)

Module-4

- 7 a. A particle moves along the curve $\vec{r} = 2t^2 \hat{i} + (t^2 - 4t)\hat{j} + (3t - 5)\hat{k}$. Find the components of velocity and acceleration at $t = 2$. (06 Marks)
- b. Find the directional derivative of $\phi = x^2yz + 4xz^2$ at $(1, -2, -1)$ along $\vec{a} = 2\hat{i} - \hat{j} - 2\hat{k}$. (05 Marks)
- c. Find $\text{div } \vec{f}$ for $\vec{f} = \nabla(x^3 + y^3 + z^3 - 3xyz)$. (05 Marks)

OR

- 8 a. Find the unit tangent vector to the curve $\vec{r} = t^2 \hat{i} + 2t \hat{j} - t^3 \hat{k}$ at $t = \pm 1$. (06 Marks)
- b. Find the unit normal vector to the surface $xy + yz + zx = c$ at the point $(-1, 2, 3)$. (05 Marks)
- c. Show that $\vec{f} = (z + \sin y) \hat{i} + (x \cos y - z) \hat{j} + (x - y) \hat{k}$ is irrotational. (05 Marks)

Module-5

- 9 a. Solve $\frac{dy}{dx} = \frac{y}{x} + \sin\left(\frac{y}{x}\right)$. (06 Marks)
- b. Solve $\frac{dy}{dx} + y \cot x = \sin x$. (05 Marks)
- c. Solve $(x^2 + y) dx + (y^3 + x) dy = 0$. (05 Marks)

OR

- 10 a. Solve $\frac{dy}{dx} = (4x + y + 1)^2$. (06 Marks)
- b. Solve $\frac{dy}{dx} + \frac{2}{x}y = \frac{3x^2 + 1}{x^2}$. (05 Marks)
- c. Solve $[y(1 + \frac{1}{x}) + \cos y] dx + (x + \log x - x \sin y) dy = 0$. (05 Marks)
